Performance Engineering
Case study: “Jacobi” stencil

The basics in two dimensions (2D)

Layer condition in 2D

From 2D to 3D

OpenMP parallelization strategies and layer condition in 3D

NT stores

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Stencil schemes

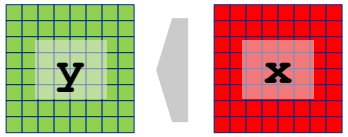
- Stencil schemes frequently occur in PDE solvers on regular lattice structures
- Basically it is a sparse matrix vector multiply (spMVM) embedded in an iterative scheme (outer loop)
- but the regular access structure allows for matrix free coding

```
do iter = 1, maxit

   Perform sweep over regular grid: y ← x

   Swap y ←→ x

enddo
```



- Complexity of implementation and performance depends on
  - update scheme, e.g. Jacobi-type, Gauss-Seidel-type,…
  - spatial (stencil) extent, e.g. 7-pt or 25-pt in 3D,…
  - …

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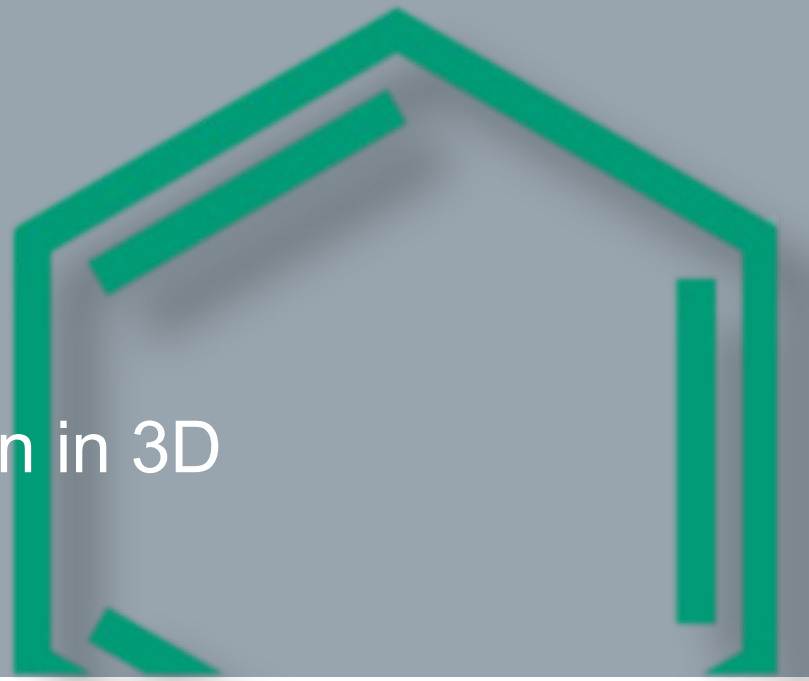


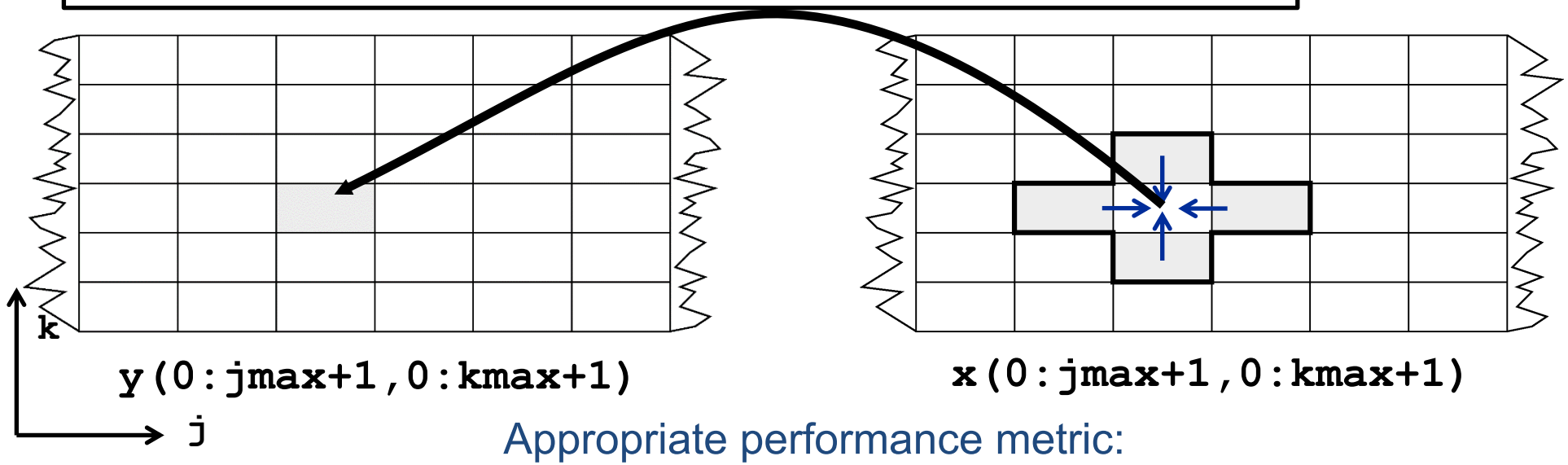
Jacobi-type 5-pt stencil in 2D

sweep

```
do k=1,kmax
  do j=1,jmax
    y(j,k) = const * (  x(j-1,k) + x(j+1,k) &
                      +    x(j,k-1) + x(j,k+1) )
  enddo
enddo
```

Lattice site update (LUP)

2 arrays

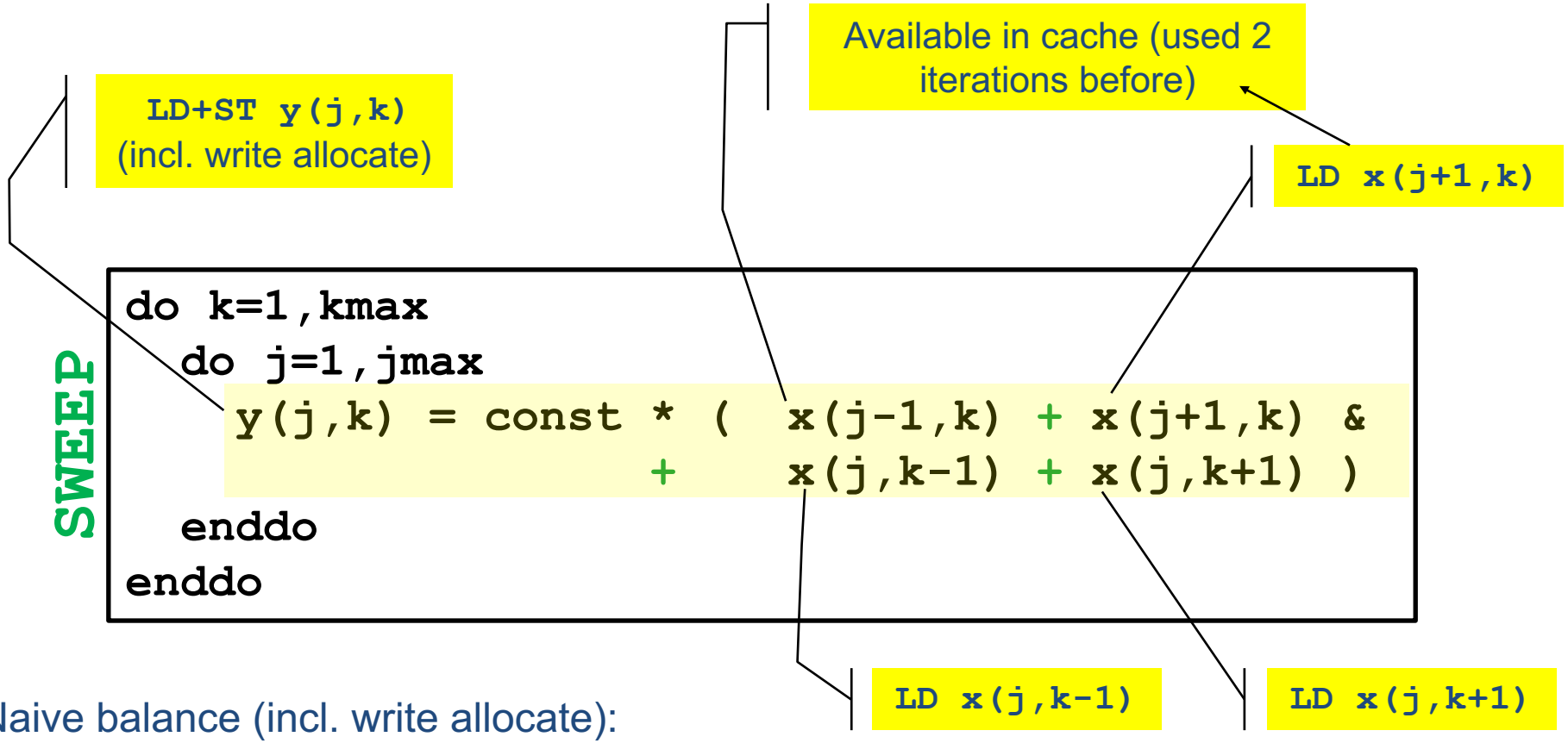


Appropriate performance metric:

"Lattice site updates per second" [LUP/s]

(here: Multiply by 4 FLOP/LUP to get FLOP/s rate)

Jacobi 5-pt stencil in 2D: data transfer analysis

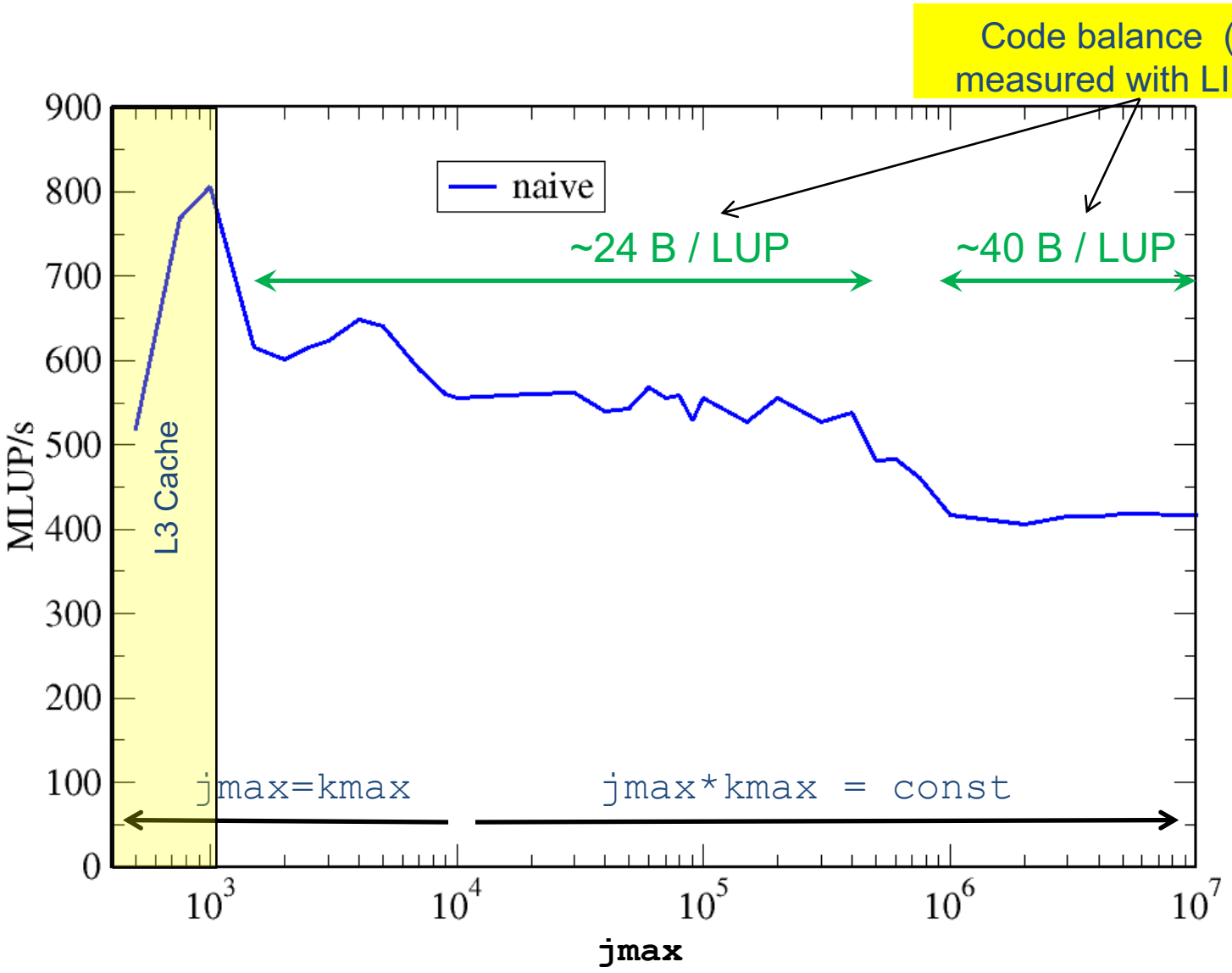


Naive balance (incl. write allocate):

`x( :, :)` : 3 LD +
`y( :, :)` : 1 ST+ 1LD

→ B_C = 5 Words / LUP → B_C = 40 B / LUP (assuming double precision)

Jacobi 5-pt stencil in 2D: Single core performance



Questions:

1. How to achieve 24 B/LUP also for large **`jmax`**?
2. How to sustain >600 MLUP/s for **`jmax > 10⁴`** ?
3. Why 24 B/LUP anyway???

Intel Compiler: ifort V13.1

Intel Xeon E5-2690 v2 ("IvyBridge"@3 GHz)

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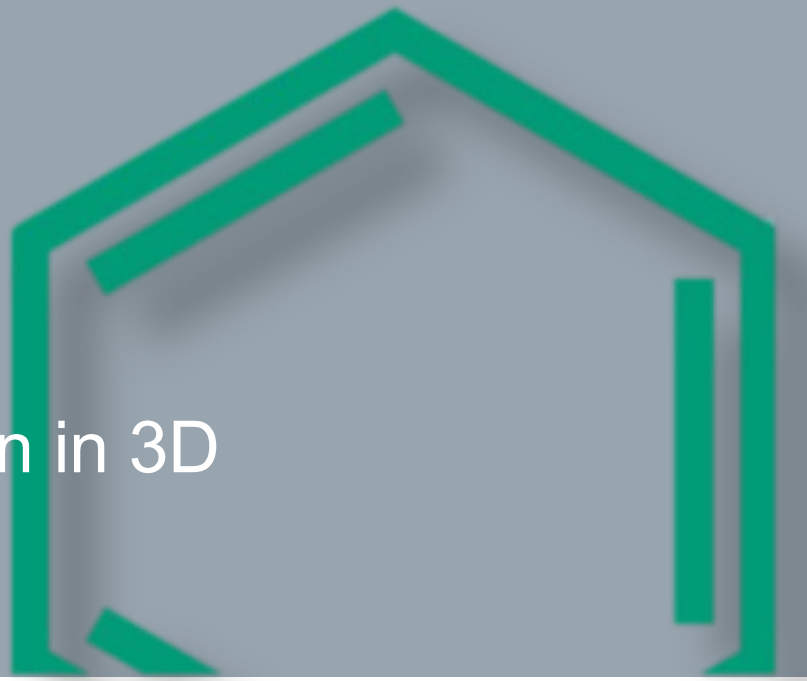
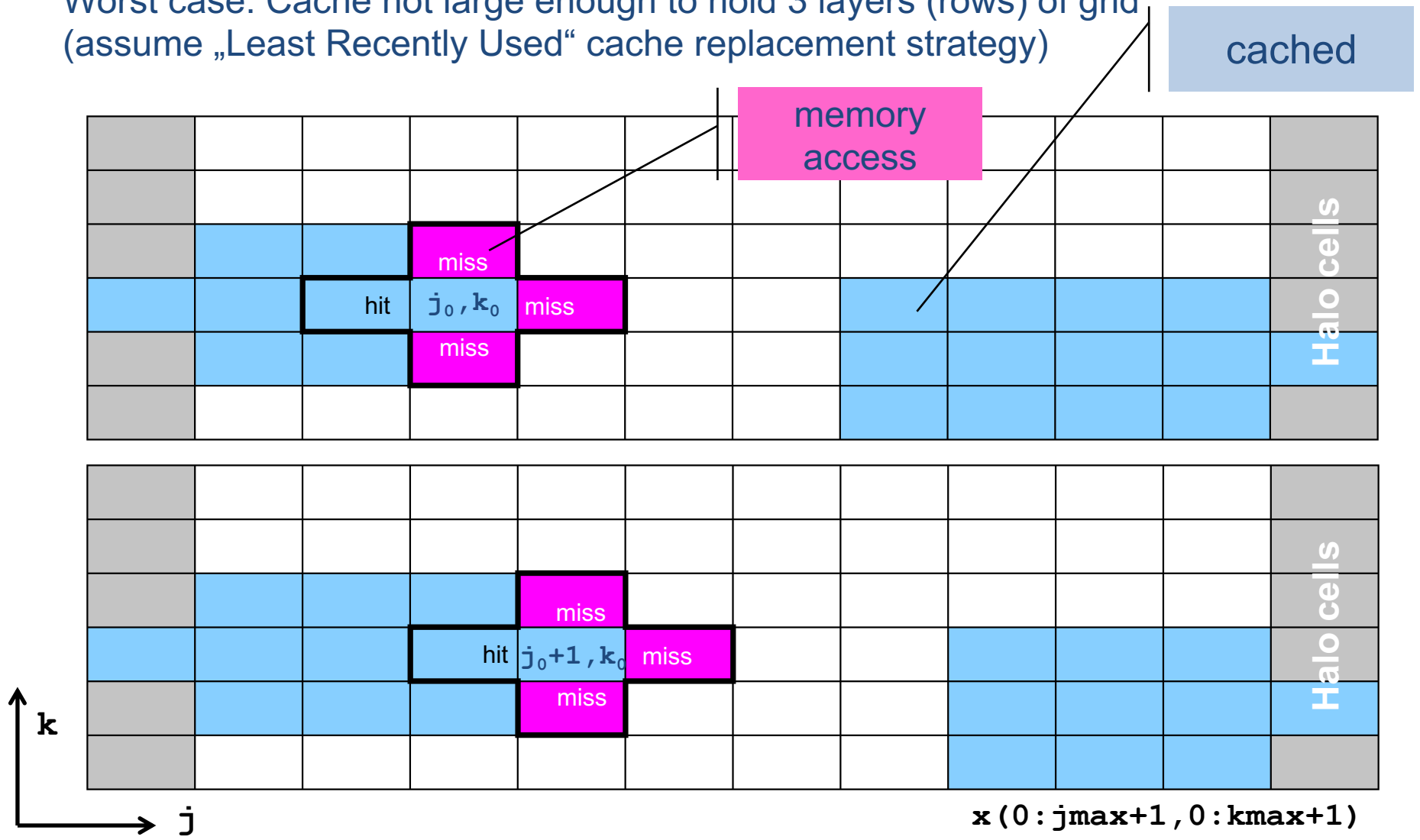


Analyzing the data flow

Worst case: Cache not large enough to hold 3 layers (rows) of grid
(assume „Least Recently Used“ cache replacement strategy)



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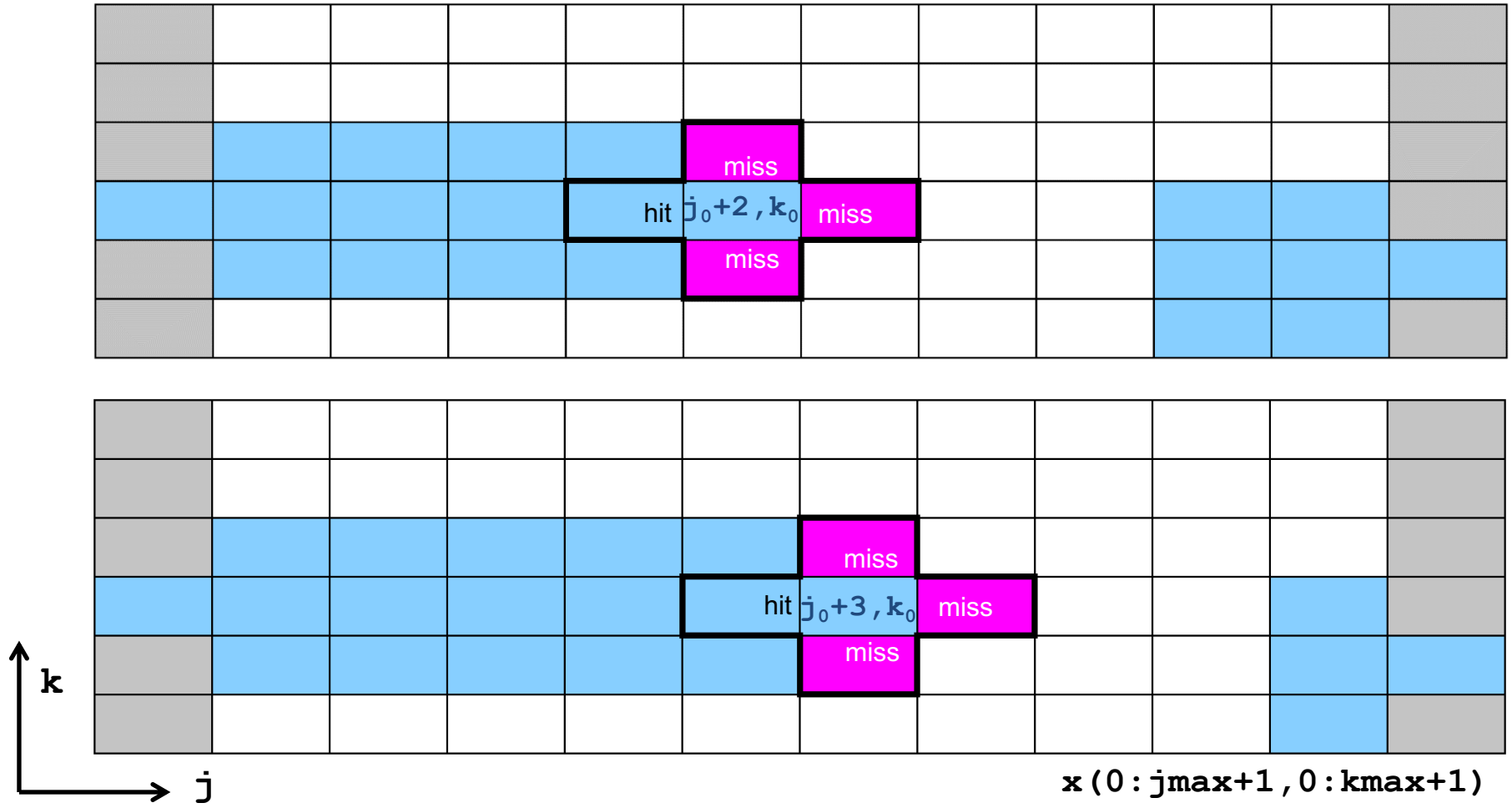
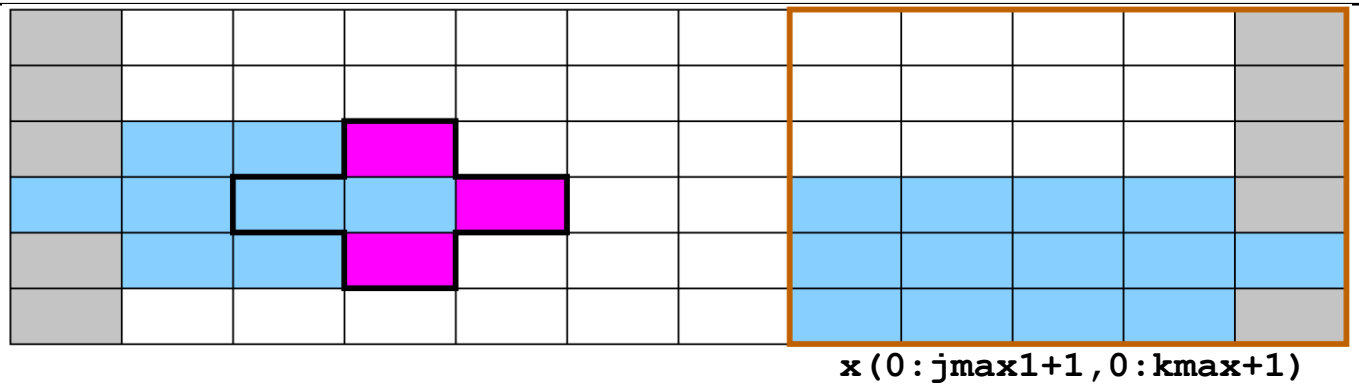
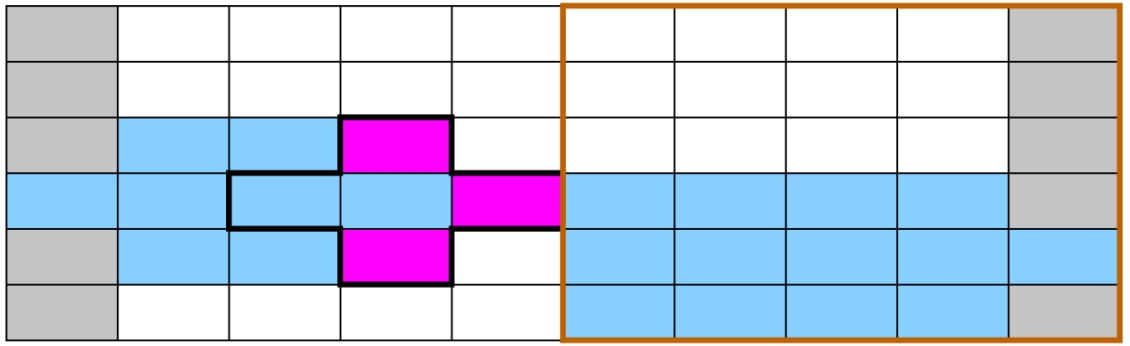


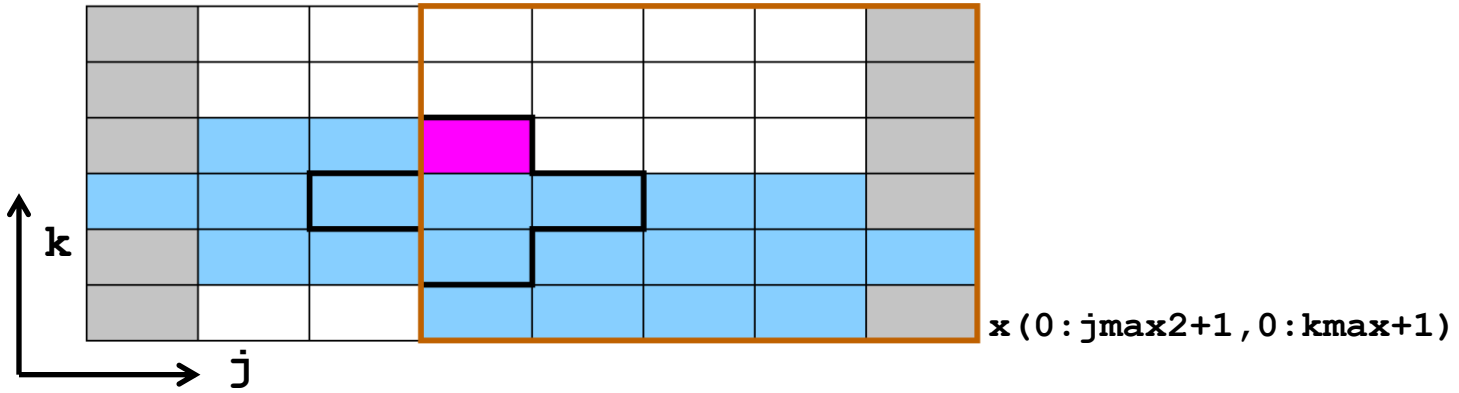
Analyzing the data flow

Reduce inner (j-) loop dimension successively



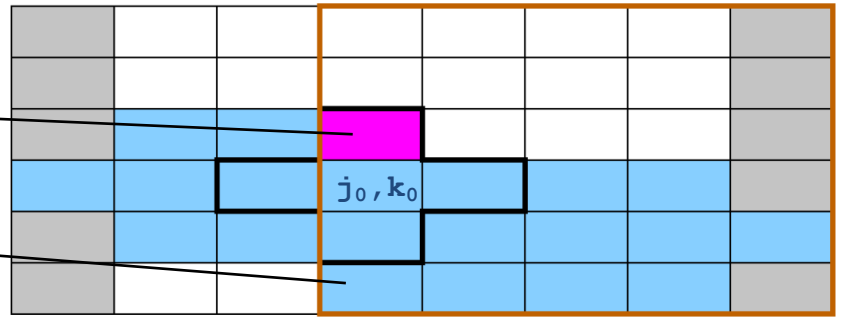


Best case: 3 „layers“ of grid fit into the cache!



Analyzing the data flow: Layer condition

2D 5-pt Jacobi-type stencil




```
do k=1,kmax
  do j=1,jmax
    y(j,k) = const * (x(j-1,k) + x(j+1,k) &
                   +   x(j,k-1) + x(j,k+1) )
  enddo
enddo
```

hit

3*jmax * 8B < CacheSize/2

"Layer condition"

3 rows of **jmax**

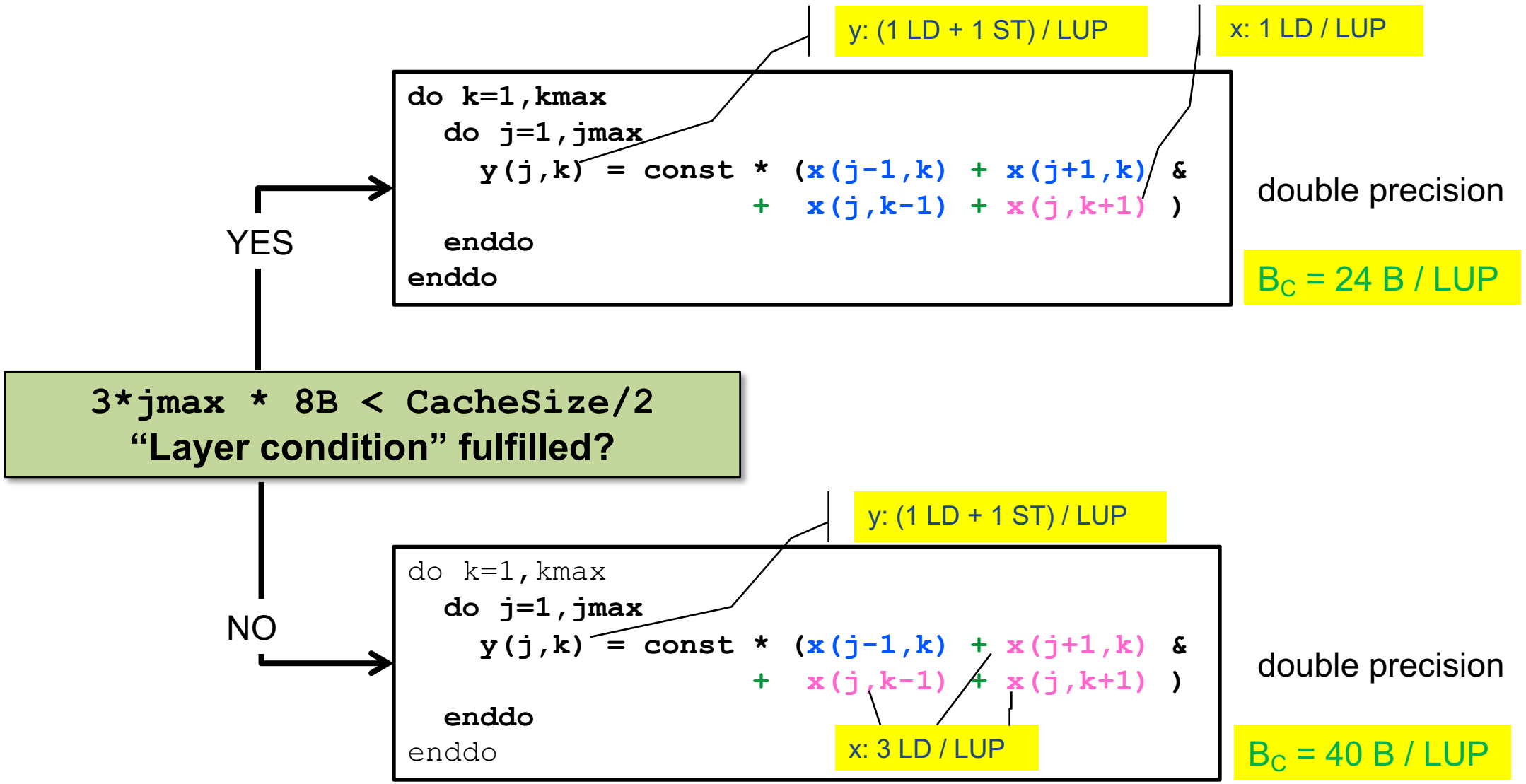
double precision

Safety margin (Rule of thumb)

Layer condition:

- No impact of outer loop length (**kmax**)
- No strict guideline (cache associativity – data traffic for **y()** not included)
- Needs to be adapted for other stencils, e.g., in 3D, long-range, multi-array,…

Analyzing the data flow: Layer condition (2D 5-pt Jacobi)



Analyzing the data flow: Layer condition (2D 5-pt Jacobi)

- How to establish the layer condition for all domain sizes ?
- Idea: Spatial blocking
  - Reuse elements of `x()` as long as they stay in cache
  - Sweep can be executed in any order, e.g., compute blocks in j-direction

→ "Spatial Blocking" of j-loop:

```
do jb=1,jmax,jblock !      Assume jmax is multiple of jblock
  do k=1,kmax
    do j= jb, (jb+jblock-1) ! Length of inner loop: jblock
      y(j,k) = const * (x(j-1,k) + x(j+1,k) &
                      +  x(j,k-1) + x(j,k+1) )
    enddo
  enddo
enddo
```

New layer condition (blocking)
`3* jblock * 8B < CacheSize/2`

→ Determine for given `CacheSize` an appropriate `jblock` value:

`jblock < CacheSize / 48 B`